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Computation, Mathematics and Logistics Department Research and Development Report

AN EVALUATION OF THE NEKTON PROGRAM

by

Bill H. Cheng Janet S. Dean Ronald W. Miller

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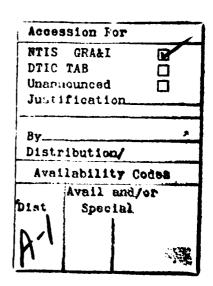
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ABSTRACT

This report describes the David Taylor Research Center's experiences with the NEKTON program, a computational fluid dynamics tool developed at the Massachusetts Institute of Technology. The theory and applications of the program are reviewed in a literature survey. A number of test cases were run successfully at DTRC and the results are promising. The applicability of NEKTON to high Reynolds number flows with three-dimensional complex geometries will require further study.

INTRODUCTION

The David Taylor Research Center's Numerical Fluid Dynamics Branch was tasked by the Office of the Chief of Naval Research (OCNR Code 1215) to evaluate the NEKTON computational fluid dynamics program. NEKTON uses an advanced numerical method to solve the Navier-Stokes equations for incompressible fluid flow problems. The program could be useful for Navy hydrodynamic design and evaluation projects. NEKTON was developed by Professor Anthony Patera and his colleagues at the Massachusetts Institute of Technology (MIT), with OCNR support [1-8].

The transition of the NEKTON code from MIT to DTRC began in March, 1990. The program was converted to an Apollo workstation and to the Cray X-MP/24 supercomputer at DTRC. A literature survey was conducted and NEKTON was exercised for several test cases in order to explore the capabilities and limitations of the program.

This report documents DTRC's experience with the NEKTON program. Promising applications to Navy problems are identified and recommendations are given.

NEKTON OVERVIEW

NEKTON is a versatile tool for solving a wide variety of fluid dynamics and heat transfer problems, including steady and unsteady incompressible flow problems in two or three spatial dimensions. NEKTON version 2.6, which was evaluated for this project, was released in February, 1990. This version of the program includes a preprocessor and postprocessor designed to make NEKTON easier to use. PRENEK is an interactive geometry generation program which aids in the preparation of the input data needed by the NEKTON computational program. POSTNEK provides an interactive plotting capability for displaying the results. The NEKTON programs were designed to run on most computer platforms having FORTRAN 77, a Unix operating system, and an X-WINDOW graphics interface.

NEKTON uses a spectral element method to solve the Navier-Stokes equations. Spectral element methods are higher-order weighted-residual techniques for solving partial differential equations. These techniques combine the geometric flexibility of finite element methods with the high accuracy of spectral methods. The method used by NEKTON differs from similar methods in the treatment of continuity conditions across element boundaries [1]. The computed functions are required to be continuous for contiguous elements.

For the spatial discretization, the physical (fluid) domain is subdivided into a number of four-sided elements in two-dimensional space (brick elements in three-dimensional space). The distribution of elements is flexible, so elements can be concentrated near regions of interest. For higher-order computations, the program automatically constructs a computational mesh for each element (see Fig. 1) with nodes corresponding to the collocation points determined by Gauss-Lobatto quadrature. For each element, both the independent variables and dependent variables are mapped onto a unit square in the computational domain using interpolation functions. These interpolation functions are represented by tensor products of Lagrangian polynomials. The dependent variables (velocity and pressure) may use the same interpolation functions (isoparametric elements) or lower-order interpolation functions (superparametric elements) as those of the independent variables (spatial coordinates).

In applying this spectral element method to Navier-Stokes equations subject to velocity and stress boundary conditions, a primitive variable approach is used. The governing equations are solved using finite differences in time and spectral elements in space. The time-dependent momentum equations are cast in a nonconservative form and the convection terms are treated explicitly. The diffusion terms are treated implicitly with interpolation functions. The continuity equation is satisfied at each time step. The resulting system of algebraic equations is solved by a conjugate gradient method. The iterative solvers used in NEKTON were designed to run efficiently on distributed-memory parallel processors such as the Intel vector hypercube used at MIT.

LITERATURE SURVEY

The MIT group led by Professor Patera has written a number of papers describing the theory and application of NEKTON [1-10]. These papers were the basis of a study of NEKTON's capabilities and limitations. The test cases described in these references were studied carefully and summarized in Table 1.

Table 1. Description of NEKTON test cases performed at MIT and documented in references 1-9.

FLOW CHARACTERISTICS	TEST CASE	R _n	REFERENCE
2-D laminar flow	channel expansion	109	[1]
2-D laminar flow	channel expansion	200	[2]
2-D laminar flow / heat transfer	flow past a cylinder	100, 200	[2]
3-D laminar flow / heat transfer	roughness element on channel wall	450	[2]
2-D Stokes flow	flow between rotating cylinders	< 1	[3]
2-D laminar flow	flow between rotating cylinders	300	[3]
2-D laminar flow / heat transfer	flow past a cylinder	20, 100, 200	[4]
2-D laminar flow / heat transfer	flow past a series of cylinders	125, 225	[5]
3-D Stokes flow / heat transfer	heat source in a furnace	< 1	[6]
2-D Stokes flow	interior flow in a wedge	< 1	[7]
3-D Stokes flow	two cylinders in a duct	< 1	[7]
2-D unsteady flow	decay of a free surface	moderate	[7]
2-D laminar flow	flow past a cylinder	100	[7]
3-D Stokes flow	spiral groove thrust bearing	< 1	[8]
2-D laminar flow	flow past a wedge in a channel	100, 1000	[8]
3-D laminar flow	appendage juncture flow	1000	[8]
2-D laminar flow	flow past a wedge in a channel	1560	[8]
2-D turbulent flow	backward-facing step	45000	[9]
2-D turbulent flow	driven cavity flow	50000	[9]

The first of these papers shows the motivation for the spectral element method by using a linear model of the one-dimensional advection-diffusion equation. This method was then applied to the full Navier-Stokes equations for two-dimensional flows. Channel expansion flow was used as the first test case as shown in Table 1. Other test cases include unsteady and steady fluid flow and heat transfer problems in two and three dimensions. The appendage juncture flow problem [8] and the viscous free surface problem [7] are of particular interest to the Navy. The work documented in these papers has largely focused on laminar calculations for Stokes flows and for moderate Reynolds number flows. Turbulent effects were ignored except in reference 9, for which a research version of NEKTON was used as a vehicle for testing RNG turbulence models.

References [1-9] address the issues of spatial discretization (spectral element formulation), solution algorithm (conjugate gradient and multigrid methods), and computer architecture (distributed-memory parallel processing). The developers of NEKTON are continuing to expand the capabilities of NEKTON by applying it to more realistic problems and improving its features.

NEKTON USE AT DTRC

NEKTON version 2.6 was installed on DTRC's Cray X-MP/24 supercomputer and on the Applied Mathematics Division's Apollo Domain network. For simple two-dimensional problems, PRENEK, NEKTON, and POSTNEK can be run on the Apollo workstations. For more complex problems, PRENEK is used on the Apollo for data generation, and the NEKTON number-crunching is carried out on the Cray. The computed results are then transferred back for plotting by POSTNEK on the Apollo. The use of an Apollo Cray station facilitates the transfer of data to and from the Cray version of NEKTON. Additional plotting capabilities were developed in order to produce printed plots for comparison with other calculations. The Apollo/Cray implementation will make NEKTON readily available to many researchers; DTRC currently has more Apollos than any other type of scientific workstation. NEKTON could also be installed on other DTRC computers if necessary.

The two sample problems described in the NEKTON users manual [10] were run for both the Apollo and Cray versions of NEKTON. The PRENEK, NEKTON, and POSTNEK programs worked flawlessly for these sample problems. As part of this initial study, procedures were written to interface the Apollo and Cray NEKTON systems. The lack of a Unix-based operating system on the Cray greatly complicated this

process. The UNICOS operating system now being installed on DTRC's Cray computer should allow a somewhat simpler interaction between the Apollo and Cray NEKTON codes.

Several additional problems were chosen to explore NEKTON'S capabilities. Relatively simple geometries were used so that several of NEKTON's important features could be studied. Test cases were chosen for which experimental data or analytic solutions exist. These test cases verify NEKTON's unsteady flow capability, the inflow and outflow boundary conditions, and a computational capability for flow separation.

The first test case is the unsteady one-dimensional Couette flow, a parallel flow in a channel with the bottom wall suddenly accelerating to a constant velocity. For this case, the momentum balance is between the local acceleration and the diffusion since the convection vanishes identically. The exact solution of this classical problem exists and can be expressed as the sum of complementary error functions [11].

NEKTON was used to compute the velocity distribution above the suddenly accelerated plane wall. As shown in Fig. 1, the physical domain contains 12 spectral elements with a 5 x 5 mesh consistent with fourth-order interpolating polynomials. The upstream and downstream boundaries are specified by a constant pressure. The time evolution of the velocity distribution as computed by NEKTON and by the exact solution is presented in Fig. 2. The agreement of the computed results with the exact solution is remarkably good.

The second test case is the steady two-dimensional flow near the inlet of a channel. The channel has two parallel plane walls separated by a distance h and a length of seven times h. The physical domain contains 42 spectral elements with a 5 x 5 mesh, as shown in Fig. 3. The no-slip condition is applied to the channel walls, a uniform inflow is specified at the inlet, and a constant pressure is specified at the downstream boundary. The initial condition is a state at rest and time evolution of the velocity is computed until a steady state is reached.

The steady state solution is displayed in Fig. 4. Computations were performed for a Reynolds number of 100 based on the channel width. The computed results show that a fully developed flow with a parabolic velocity distribution is reached near the downstream boundary. This observation compares well with the boundary layer calculations in [11]. Figs. 5 and 6 give the maximum velocity and the wall shear stress as a function of the longitudinal location, with the corresponding results for the fully developed flow as the baseline.

The third case is the steady two-dimensional flow over a backward facing step. The physical domain with a 5 x 5 spectral element mesh is shown in Fig. 7. The inflow boundary is specified by a parabolic velocity distribution and a constant pressure is applied at the outflow boundary. Computations were performed for a Reynolds number of 150. The computed streamlines in Fig. 8. show flow separation and compare favorably with the experiments in [12]. The computed separation zone has the right length, as compared with the experiments. Even closer agreement was obtained in comparison with similar calculations using the DTNS2D code [13].

A comparison of the DTNS2D and NEKTON computational grids shows that NEKTON allows a relatively coarse grid for similar accuracy. This is attributed to the higher-order spatial discretization within a spectral element. NEKTON also computes the time development of the separation zone at each time step. DTNS2D is currently limited to steady solutions, as are most finite-difference and finite-element RANS codes. NEKTON work has so far focused on moderate Reynolds number flows, while the steady DTNS3D code has been applied successfully to high Reynolds number flows with complex submarine geometries [14]. Unsteady DTNS2D and DTNS3D codes are under development at this time.

LIMITATIONS

NEKTON version 2.6 has a number of limitations which would impact its usefulness for Navy projects. Some of these limitations will be removed in future releases of NEKTON. For example, turbulence modeling is not currently included in NEKTON, but Professor Patera has indicated that an algebraic turbulence model will be added in late 1990. The use of NEKTON for realistic problems is also hampered by the lack of a multigrid solver in NEKTON 2.6. Inclusion of a multigrid capability in future releases of NEKTON should speed up the computations for complex geometric models.

Improvements in NEKTON'S data generation capabilities would be helpful for large-scale three-dimensional problems. Currently, PRENEK generates three-dimensional data by connecting two-dimensional slices. The preparation of these two-dimensional slices would be easier if interfaces linked PRENEK to existing ship geometry generation programs and to existing ship data bases. Currently the only such link is to finite element data generators using the IDEAS data format.

NAVY APPLICATIONS FOR NEKTON

One of the objectives of the current study is to identify potential applications for the NEKTON program. Several Navy related problems are listed in this section.

- 1. Cavity Flow: Cavity flow problems related to submarine launch and recovery operations could be studied using NEKTON. Typically, these submarine operations involve a three-dimensional cavity and a shutter door (Fig. 9) for launching and recovering weapons and unmanned underwater vehicles (UUV). The flow field inside the cavity is inertia driven and is important for designing launch and recovery systems. An earlier NEKTON study of a 2-D driven-cavity flow for a Reynolds number of 50,000 [9] could serve as a model for submarine cavity flow problems.
- 2. Appendage Juncture Flow: NEKTON could be useful for evaluating submarine sail fillet designs in order to reduce hydroacoustic signatures. The flow about the appendage juncture produces a three-dimensional horseshoe vortex. An idealized appendage juncture flow for a circular cylinder intersecting a wall has been analyzed using NEKTON [8]. It is desirable to perform similar computations for a more realistic configuration and compare computed results with corresponding experiments.
- 3. Chaotic Motions: Flow circulation in a closed container with a rotating lid was computed using NEKTON in a joint effort between DTRC and MIT in FY89. This work will be extended in FY91 to Reynolds numbers above the range of experimental data. The time history of the vertical velocity at several fixed points in space will be computed. This time series data will be examined by computing power spectrum and bispectrum. These techniques will be used to investigate chaotic motions exhibited by nonlinear fluid systems.
- 4. Thermal Wake: A surface ship's thermal wake contributes to its nonacoustic signature. The passage of a ship (in particular, its stern) creates an upwelling flow of cold water in a stratified ocean. This cold water contrasts sharply with the surrounding warm water near the ocean surface and can be detected as an infrared signature. The fluid flow / heat transfer option of NEKTON can be used to analyze the thermal wake of a surface ship.

- 5. Counterrotating Bearings: Navy propeller design projects could benefit from an extension of a previous NEKTON study of the flow between rotating cylinders [3]. For this NEKTON analysis, the outer cylinder was kept at rest, while the inner cylinder was rotated at a constant angular velocity. This work could be extended to two counterrotating cylinders as found in lubrication problems. Navy applications would include the counterrotating propellers used for integrated electric drive propulsion.
- 6. Material Processing: NEKTON could be useful for Navy material processing problems.
 Computational Fluid Dynamics has not been widely applied to material science and the potential payoff is largely untapped at this time. CFD could be used to determine and control the fluid flow in material processing in order to ensure uniform texture of materials involving phase changes. In FY88, Professor Patera was funded by OCNR to study crystal growth and weld pool problems using NEKTON. This work could be extended to other Navy problems such as spray deposition.

CONCLUSION

An evaluation was performed for the NEKTON program developed by Professor Patera of MIT. A literature survey revealed impressive progress in spatial discretization, solution algorithm, and parallel processing techniques. NEKTON and its preprocessor and postprocessor were implemented on workstations and the Cray X-MP at DTRC. Test cases with simple geometries were successfully run at DTRC and the results show that NEKTON holds much promise as a hydrodynamics analysis tool.

NEKTON is now available for use at DTRC. Applications to realistic Navy problems with high Reynolds number flows and complex three-dimensional geometries are needed to better determine the capabilities and limitations of the code.

ACKNOWLEDGMENTS

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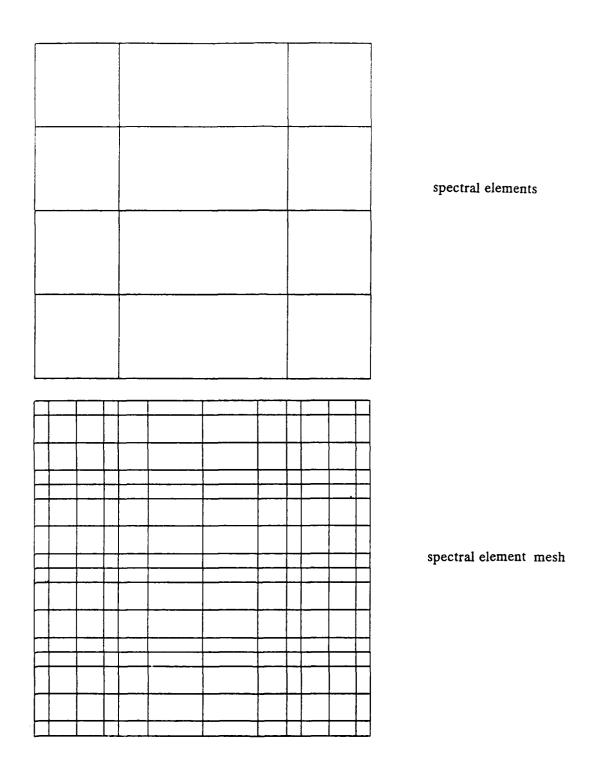


Fig. 1. A unit square subdivided into 12 spectral elements with a 5 x 5 mesh

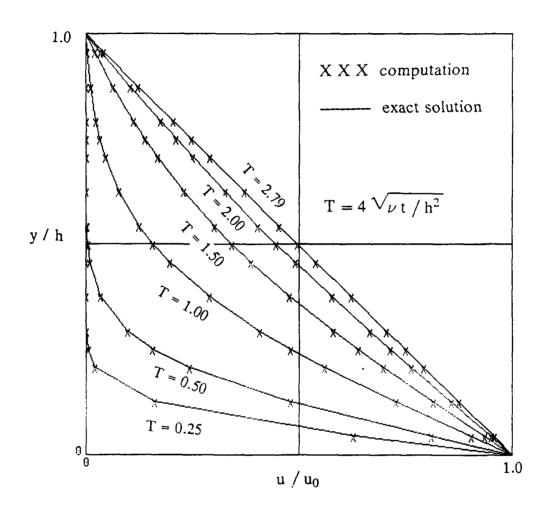


Fig. 2. Comparison of velocity distributions at different times for unsteady Couette flow

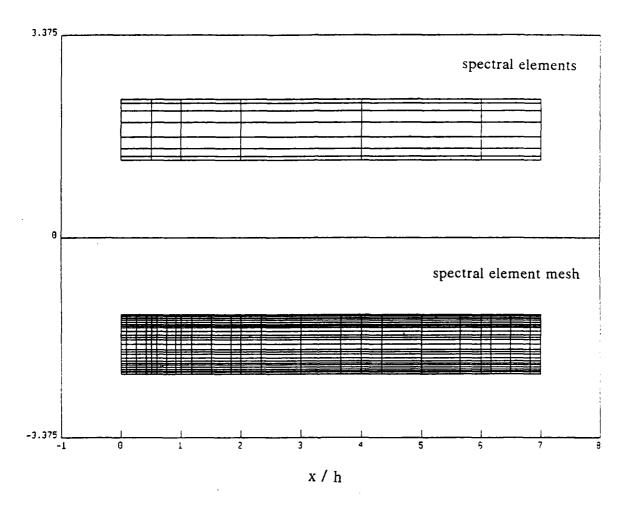


Fig. 3. Computational grid with 42 spectral elements for the inlet flow

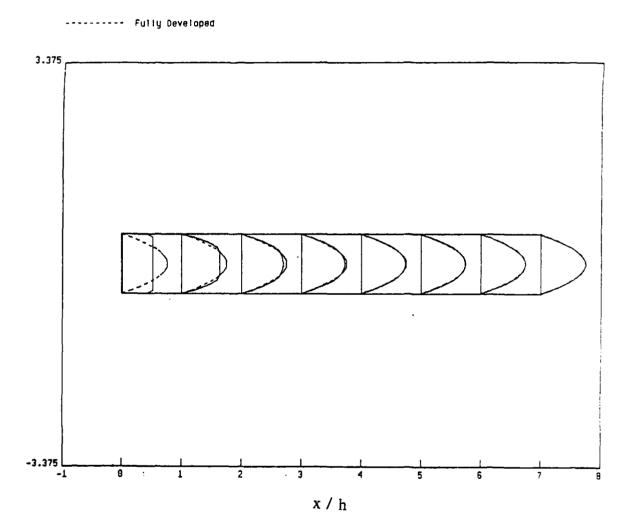


Fig. 4. Velocity distribution for the inlet flow at different locations

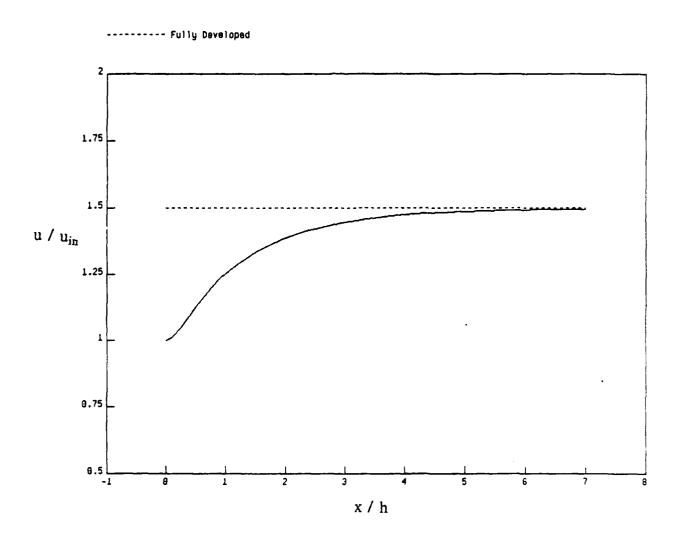


Fig. 5. Maximum velocity as a function of longitudinal location for the inlet flow

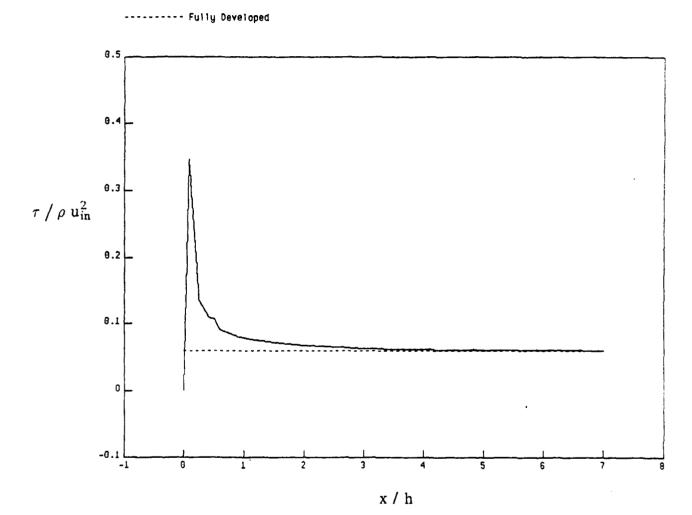


Fig. 6. Wall shear stress as a function of longitudinal location for the inlet flow

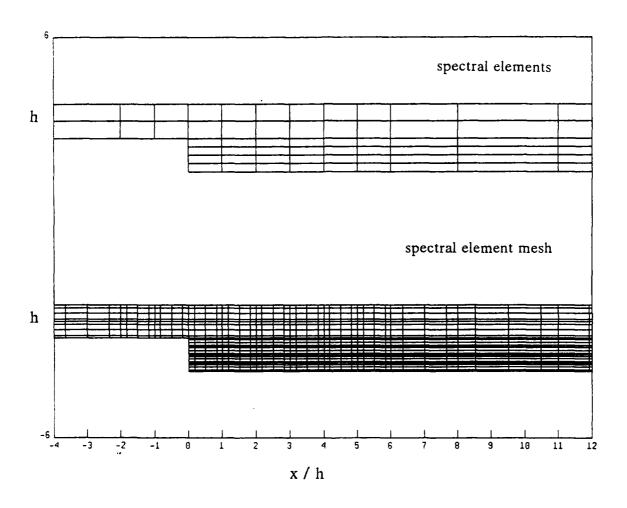


Fig. 7. Computational grid with 60 spectral elements for a backward-facing step

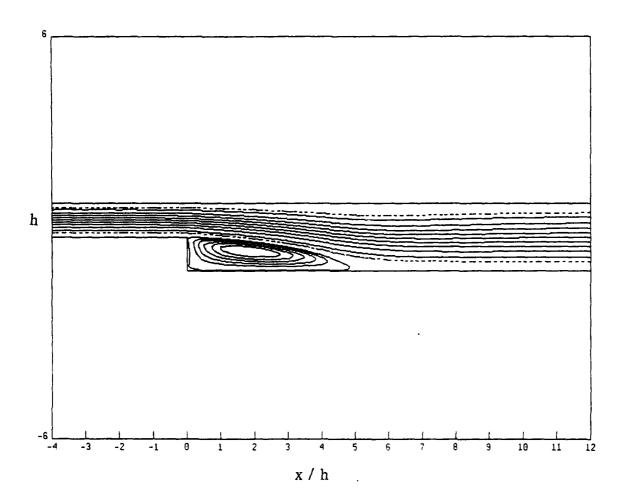


Fig. 8. Computed streamlines for the backward facing step

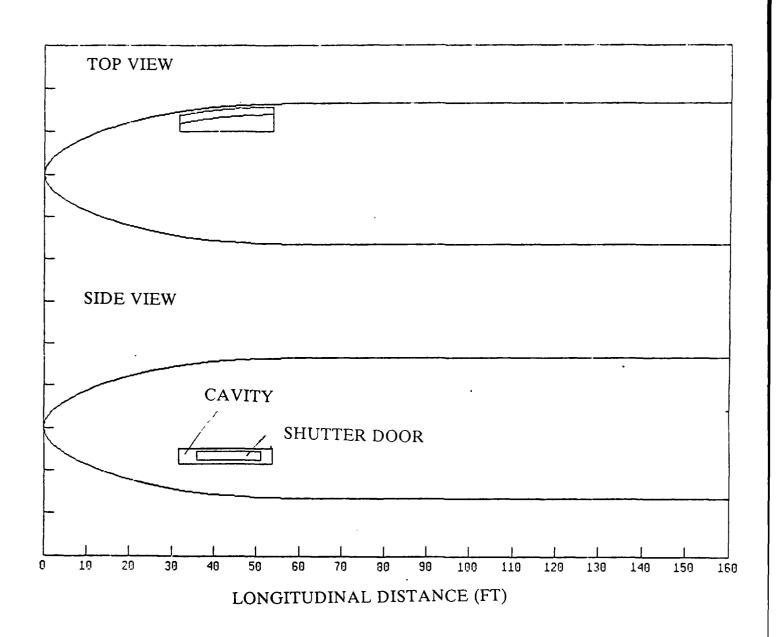


Fig. 9. Top and side views of a submarine forebody showing the cavity and shutter door

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